Reactance and Resonance

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Some Review of Important Concepts

- AC waves have constantly changing voltage and currents.
- We need to use RMS voltage and RMS currents in calculations involving AC.
- The two fundamental principles of electronics:
  - Moving electrons create magnetic fields.
  - Changing magnetic fields cause electrons to move.

Resistance and Reactance

- Resistance is one of the fundamental components of electricity. It inhibits the flow of electrons.
- Inductors and capacitors react differently under DC and AC conditions:
  - Inductors offer 0 resistance in DC environment
  - Capacitors offer infinite resistance in DC environment
- They react quite differently in an AC environment
- **The opposition to electron flow in an AC environment by inductors and capacitors is called REACTANCE.**

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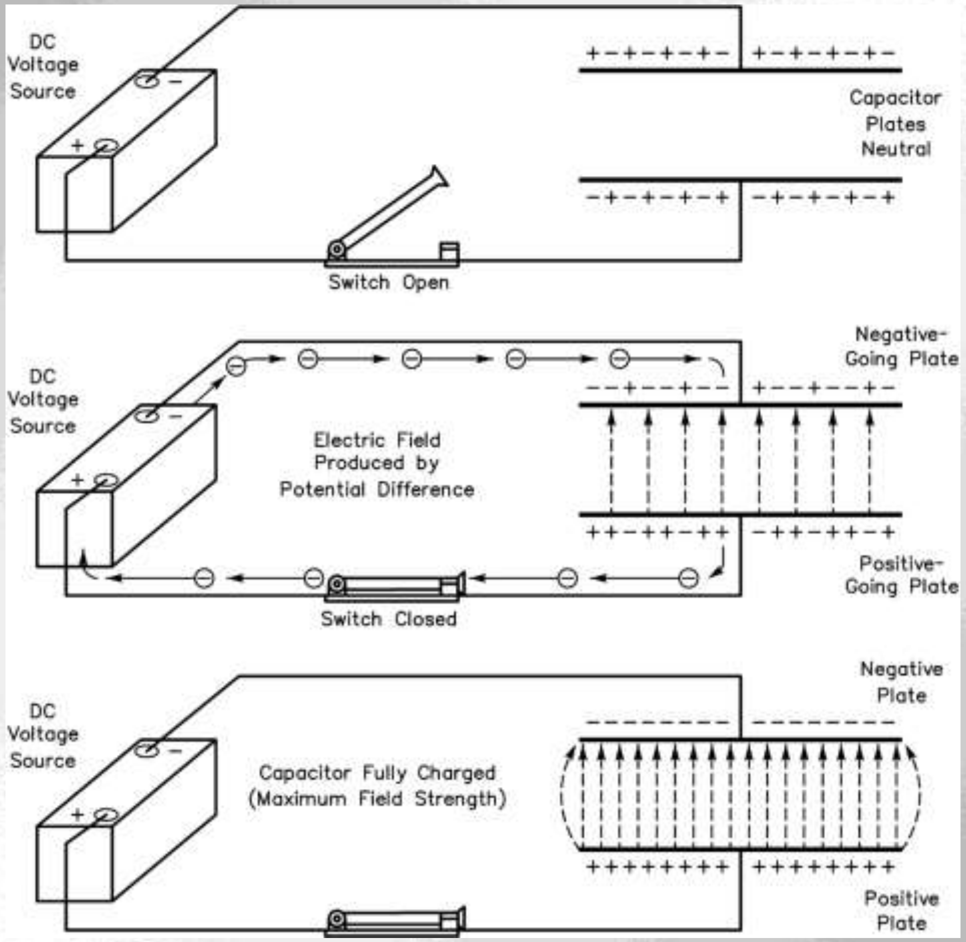
Charging a Capacitor with DC



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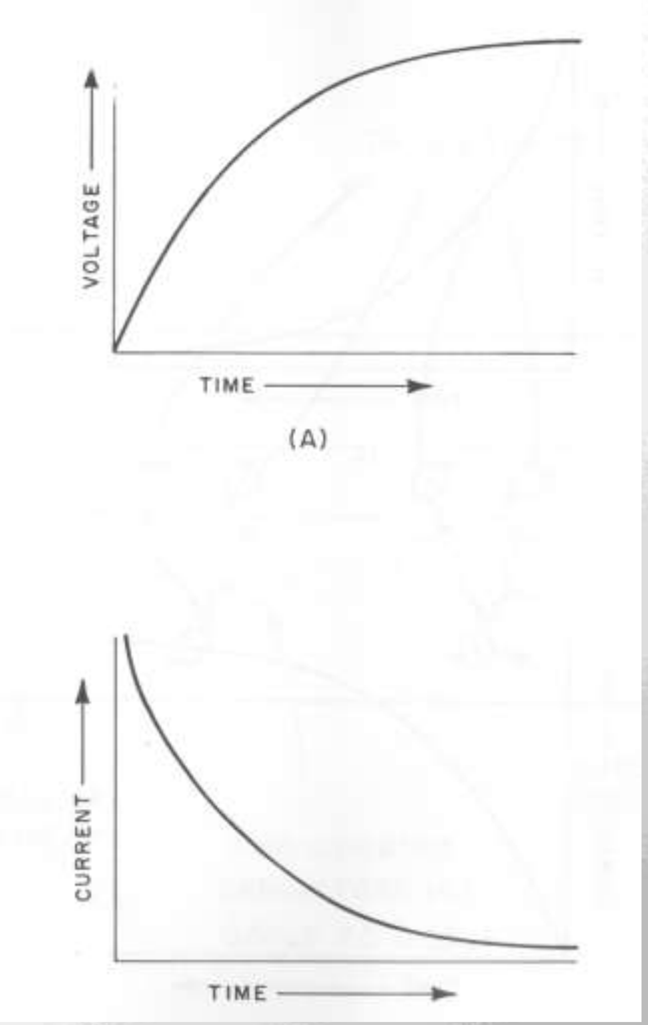
Capacitor Voltage versus Time

- At time = 0 when voltage is first applied:
  - I is very high (electrons rushing in).
  - V is 0 (capacitor looks like a short circuit).
- At some later time after voltage is applied:
  - I is zero (capacitor will accept no more electrons).
  - V is high (capacitor is fully charged).

Capacitor Voltage and Current Curves



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Capacitors Charge over Time

- As the capacitor accepts a charge, there is an opposing voltage in the capacitor.
- The increasing charge opposes a change in the voltage across the capacitor.
- Current is decreasing while voltage is increasing.
- Thus the voltage across a capacitor lags behind the current in a capacitor.

(These two concepts will become important later in the discussion of resonance)

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Capacitive Reactance

- If we apply a constant AC current in series with a capacitor and measure:
  - RMS voltage across the capacitor = **E**
  - RMS current through the capacitor = **I**
- We find that:
  - Voltage varies with the frequency.
  - Voltage varies with the value of capacitance.
- We can use Ohm's law to compute the reactance in the circuit: $X_C = E/I$.

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Capacitive Reactance Important Points

- Capacitors have more reactance at lower frequencies; the reactance decreases as frequency increases.
- The relationship between frequency and capacitive reactance is not linear.
- The formula to calculate capacitive reactance is:

$$X_C = \frac{1}{2\pi f C}$$

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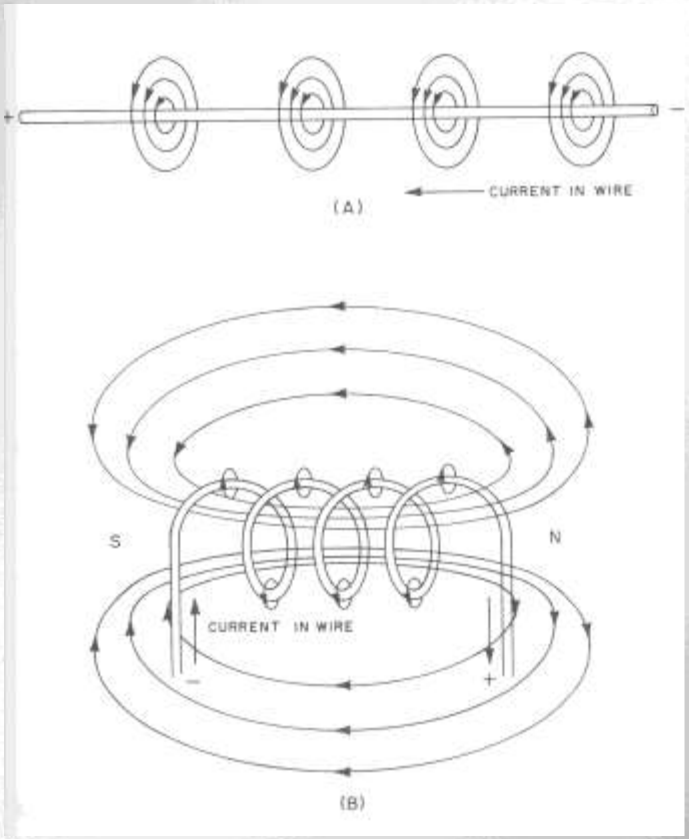
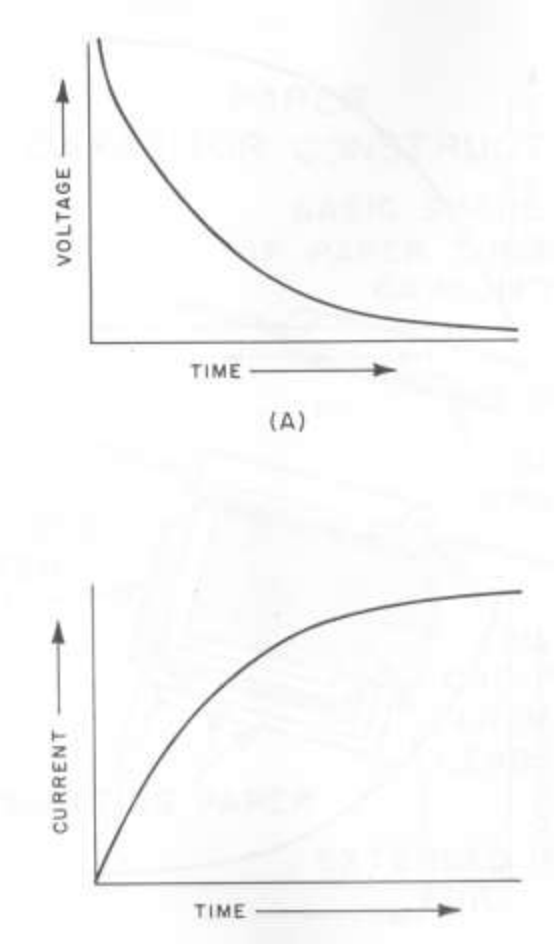
Inductors in AC Environment



Inductor Voltage and Current

- At time = 0 when voltage is first applied:
  - Voltage across inductor is very high.
  - Current through inductor is 0.
- At some later time after voltage is applied:
  - Current through inductor is maximum.
  - Voltage across inductor is very low.

Inductor Voltage and Current Curves



Inductor Current versus Time

- As the current builds up in the inductor, a magnetic field develops which opposes the flow of additional electrons .
- The changing magnetic field causes Inductors to oppose a change in current.
- The voltage across an inductor leads the current in the inductor (the opposite of what happens in a capacitor).

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Inductive Reactance

- If we apply a constant AC current in series with an inductor and measure:
  - RMS voltage across the inductor = **E**
  - RMS current through the inductor = **I**
- We find that:
  - Voltage across the inductor increases directly with frequency.
- We can use Ohm's law to compute the reactance in the circuit $\mathbf{X_L = E/I}$.

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Inductive Reactance

- Inductors have less reactance at lower frequencies; the reactance increases as frequency increases (opposite of capacitors!).
- The relationship between frequency and inductive reactance is linear.
- The formula to calculate inductive reactance is:

$$X_L = 2\pi f L$$

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Putting Inductors and Capacitors Together

- Remember: Inductors and capacitors react to changing currents (AC).
  - Voltage lags current in capacitor.
  - Voltage leads current in an inductor.
- Putting inductors and capacitors together creates an electronic “bell” that rings or resonates.
  - Inductor feeds capacitor: capacitor feeds inductor in a back and forth, reciprocal way.

Resonance

When the reactance of the inductor equals the reactance of the capacitor, the reactances cancel . This is called RESONANCE.

- If the capacitor and inductor are in series,
  - Opposition to the resonance frequency is minimum.
  - Opposition to other frequencies will be higher.
- If the capacitor and inductor are in parallel,
  - Opposition to the resonance frequency is maximum.
  - Opposition to other frequencies will be lower.

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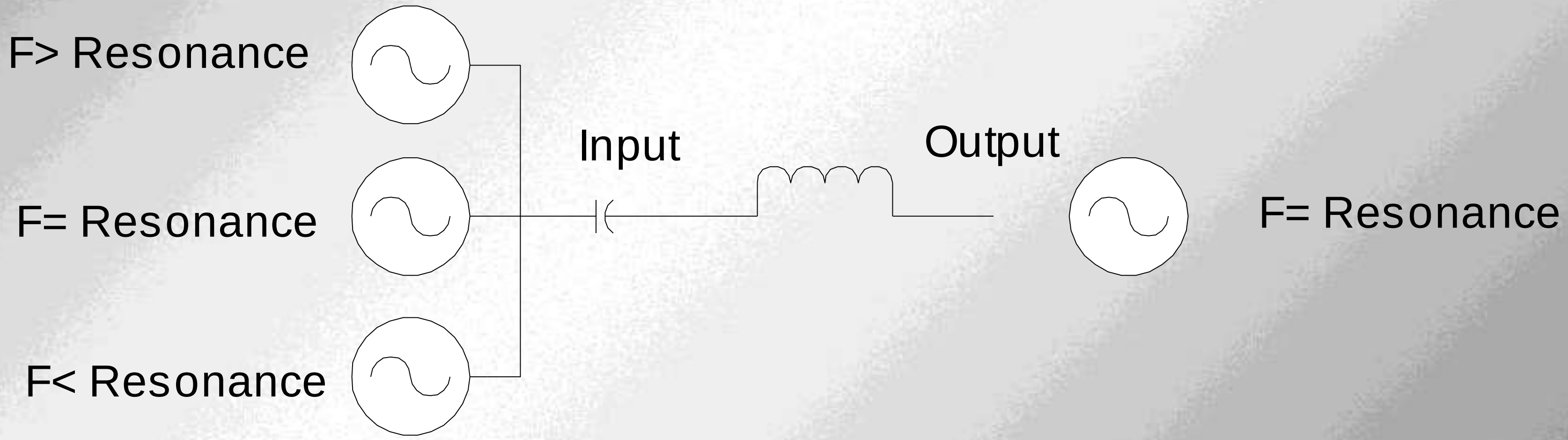
Series
Passes Resonant Frequencies



Current at Series Resonance

Remember:

1. Voltage in an inductor leads inductor current.
2. Voltage in a capacitor lags capacitor current.
3. In series circuit current is same in both components.

- At resonance – voltages are equal and opposite so they cancel out.
- Ohm's law: **R = E/I**.
- Resistance is theoretically zero because **E = 0**.
- Practical components always have some resistance so R is never zero.

A Better Explanation

- Inductive Reactance is the opposite of Capacitive Reactance.
- X_L is always positive. X_C is always negative. $X_L = 2 * Pi * F * L$, $X_C = - 1 / (2 * Pi * F * C)$
- These are conventions which come from Circuit Analysis using Calculus.
- Combine reactances in series like resistors in series. $X_T = X_L + X_C$
- When magnitudes of X_L and X_C are equal, $X_T = 0$

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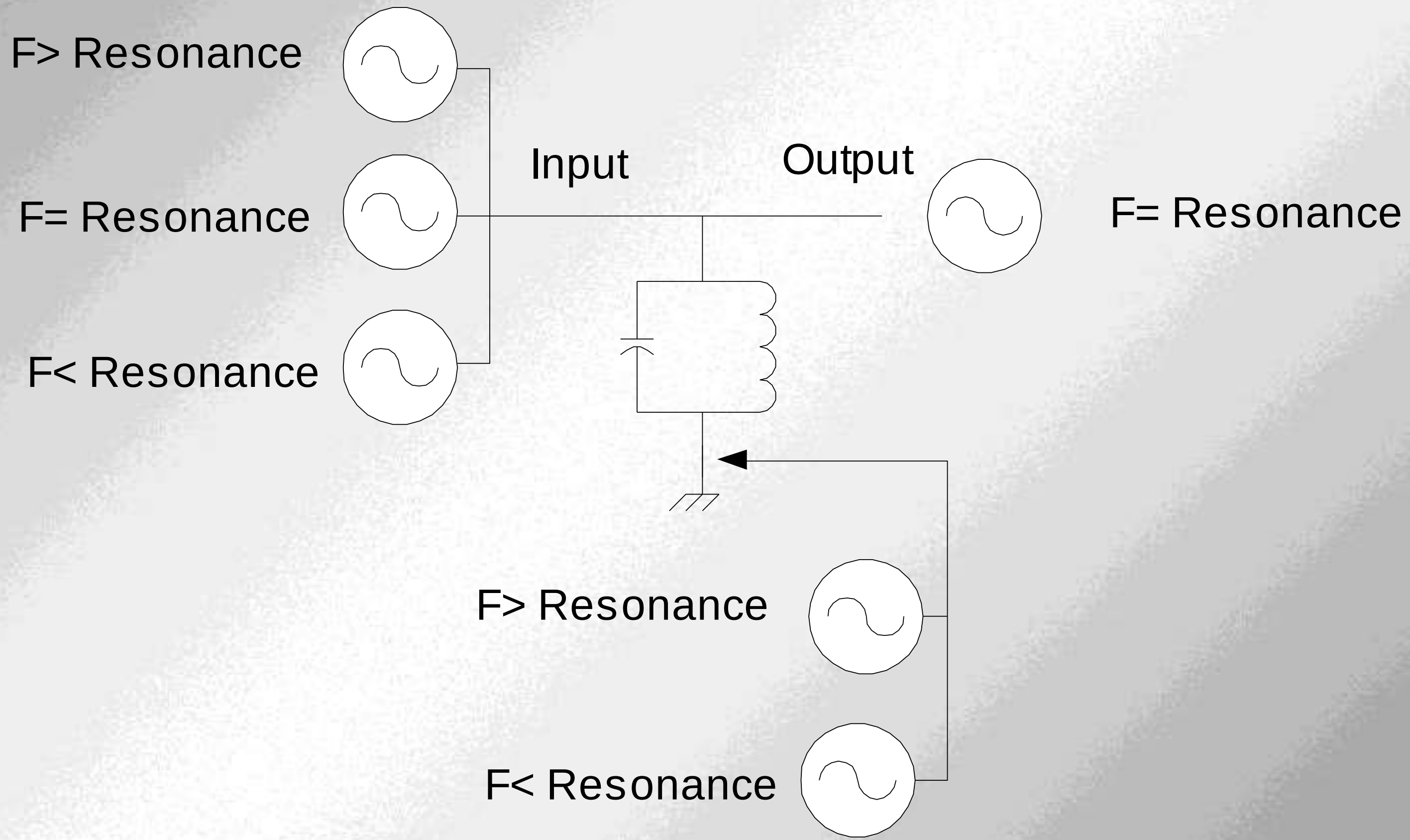
Parallel

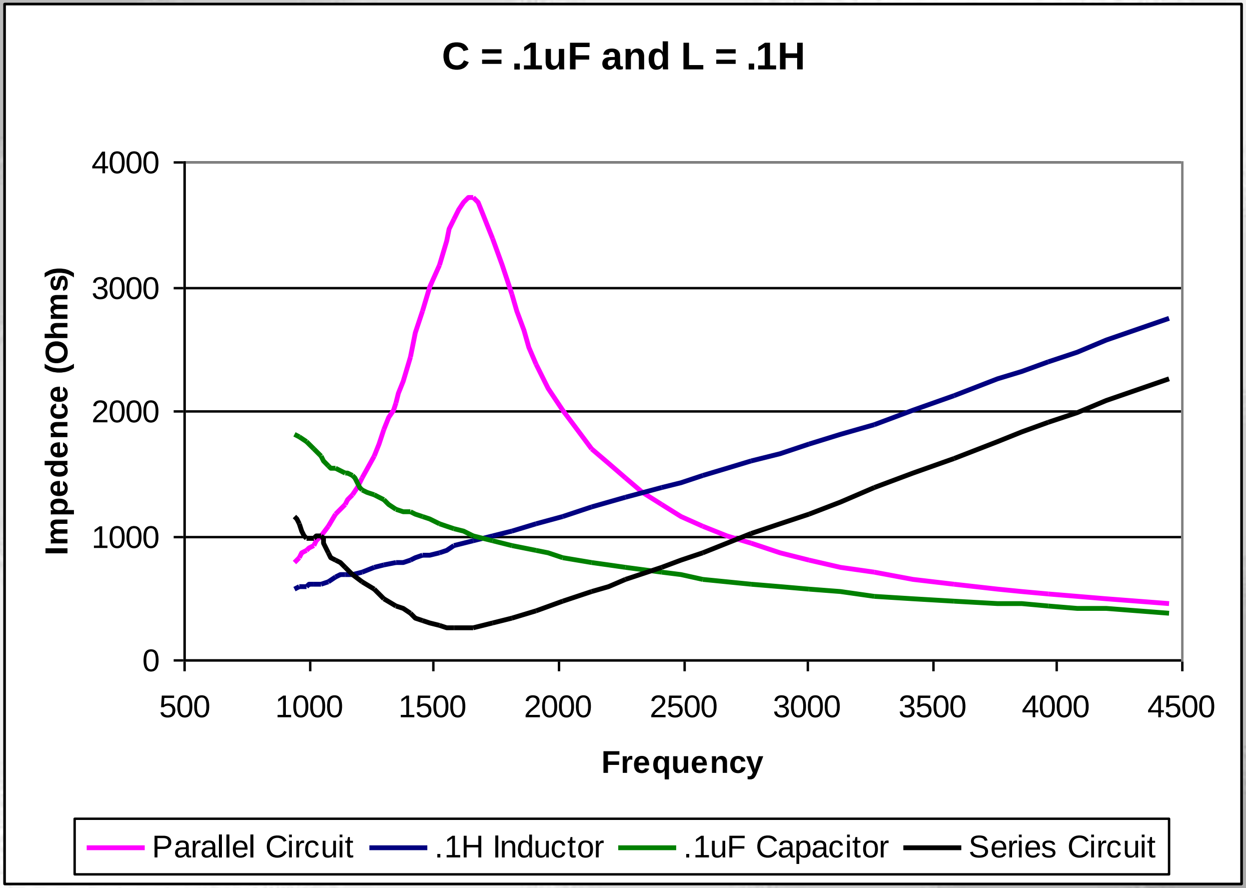
Short Circuits Non-Resonant Frequencies



Voltage at Parallel Resonance

- Remember:
  1. Voltage in inductors lead current.
  2. Voltage in capacitors lag current.
  3. In parallel circuits voltage is same in both components.
- At resonance – currents are equal and opposite so they cancel out.
- Ohm's law: **R = E/I**
- Resistance is theoretically infinite because **I = 0**
- Practical components have losses in parallel, due to leakage, which reduce the total impedance.

Putting It All Together



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Resonance Formula

- Resonance occurs when the magnitude of the capacitive and inductive reactance are equal:

$$2\pi f L = \frac{1}{2\pi f C}$$

Resonance Formula Derivation

1. $2\pi fL = \frac{1}{2\pi fC}$

2. $(2\pi fL)(2\pi fC) = 1$

3. $(2\pi)^2 f^2 LC = 1$

4. $f^2 = \frac{1}{(2\pi)^2 LC}$

5. $\sqrt{f^2} = \sqrt{\frac{1}{(2\pi)^2 LC}}$

6. $f = \frac{\sqrt{1}}{\sqrt{(2\pi)^2}\sqrt{LC}}$

7. $f = \frac{1}{2\pi\sqrt{LC}}$

Impedance Matching with L/C Circuits

- As stated in a previous lesson, maximum power is transferred if the impedances between stages are matched.
- The reactance characteristics of L/C circuits can be used as impedance transformers to create a match.
  - Example: L/C circuits are used as RF impedance transformers in antenna matching networks.

Important Stuff

- AC voltages measured in Peak-to-Peak and RMS.
- RMS is the effective voltage for AC.
- Inductors and capacitors react to AC differently than DC (Opposition to AC is called Reactance).
- Capacitive reactance, X_C, goes down as frequency goes up.
- Inductive reactance, X_L, goes up as frequency goes up.
- An L/C circuit is resonant at a frequency where the inductive and capacitive reactances are equal.